Introduction to Interferometry

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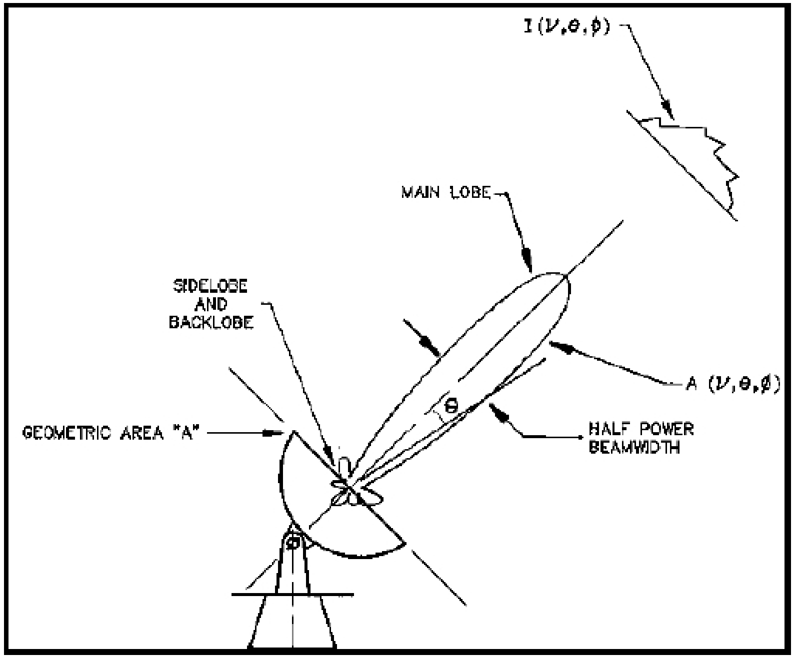
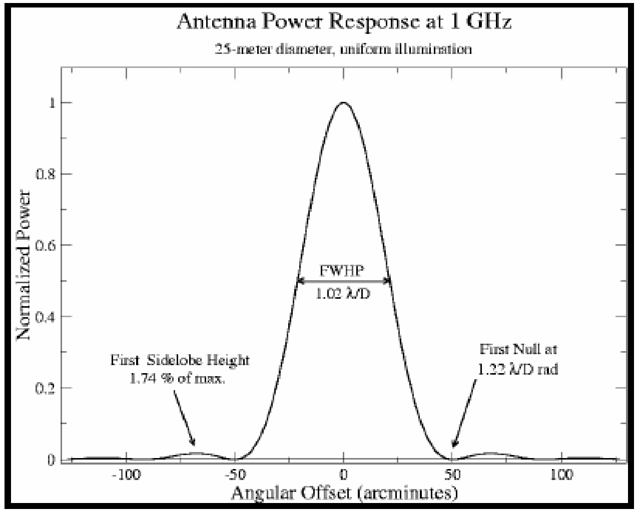
Single Antenna

The Receiving Power has an Airy Diffraction pattern!





Note the mainlobe, sidelobes and backlobes of the power pattern.

The FWHM of the mainlobe of the power pattern is (D: the diameter of the antenna aperture)

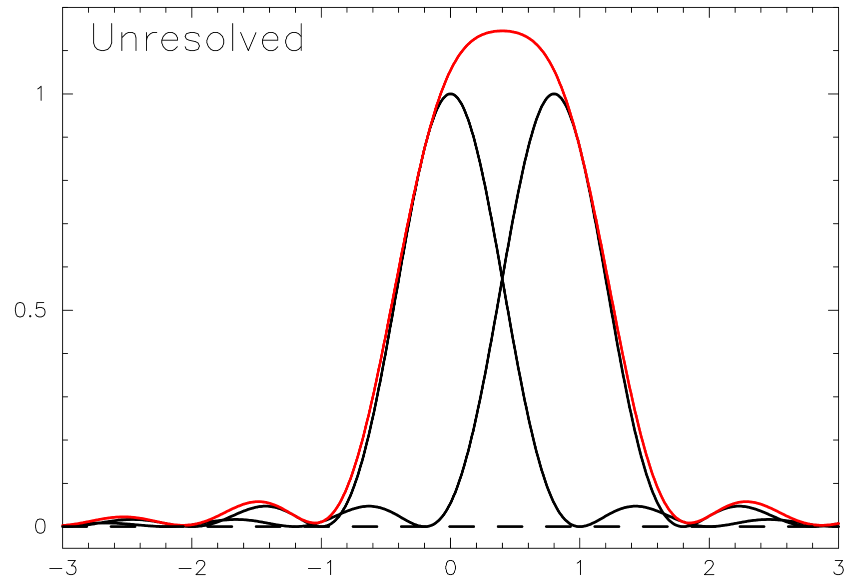
$$\mathrm{FWHM} = 1.02\frac{\lambda}{D}\mathrm{rad} \sim 58.4^\circ\,\frac{\lambda}{D} \qquad \text{with} \quad \lambda = \frac{c}{\nu}$$

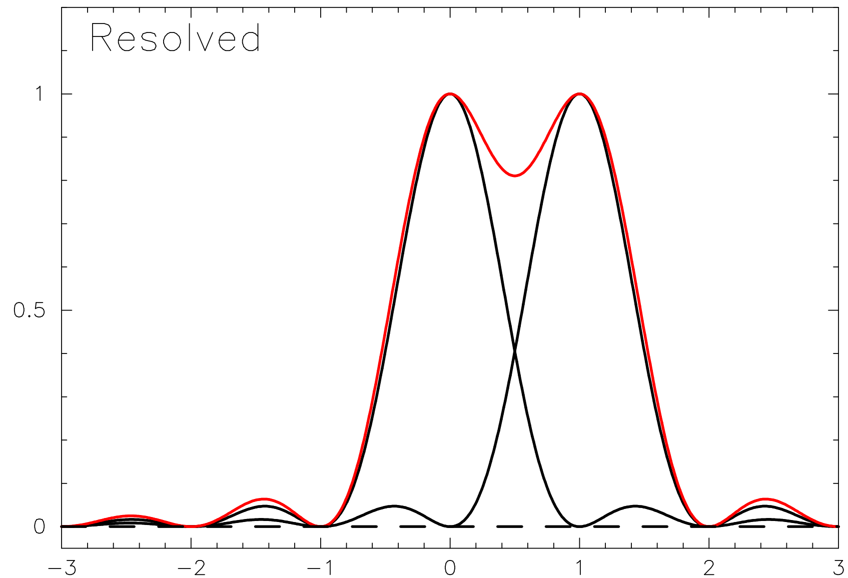
This FWHM is the angular resolution of your map, when scanning over the sky with this antenna!

To be strict, use $1.22\frac{\lambda}{D}$, the first null, for the resolution! What about ALMA antenna 12 m in diameter at 100 GHz? (1 radian $\approx$ 206265 arcsec).

Angular Resolution

Angular resolution: it is the smallest angular separation which two point sources can have in order to be recognized as separate objects.





The Quest for Angular Resolution

Angular resolution: it is the smallest angular separation which two point sources can have in order to be recognized as separate objects.

The Quest for Angular Resolution

Angular resolution of a single antenna is $\theta \sim \lambda/D$

The 30-meter aperture of a e.g., IRAM antenna provides a resolution, e.g., $\sim$ 20.6 arcsecond at 100 GHz ($\lambda = 3$mm), too low for modern astronomy!! (Note 1 radian $\approx$ 206265 arcsec).

How about increasing D i.e., building a bigger telescope? This trivial solution, however, is not practical. e.g., 1" resolution at $\lambda = 3$ mm requires a 600 m aperture!! Larger for longer wavelength!!

Green Bank Telescope (100 m) (Left) 1988.11.15 (Right) 1988.11.16

Aperture synthesis

Aperture synthesis: synthesizing the equivalent aperture through combinations of elements, i.e., replacing a single large telescope by a collection of small telescope “filling" the large one!

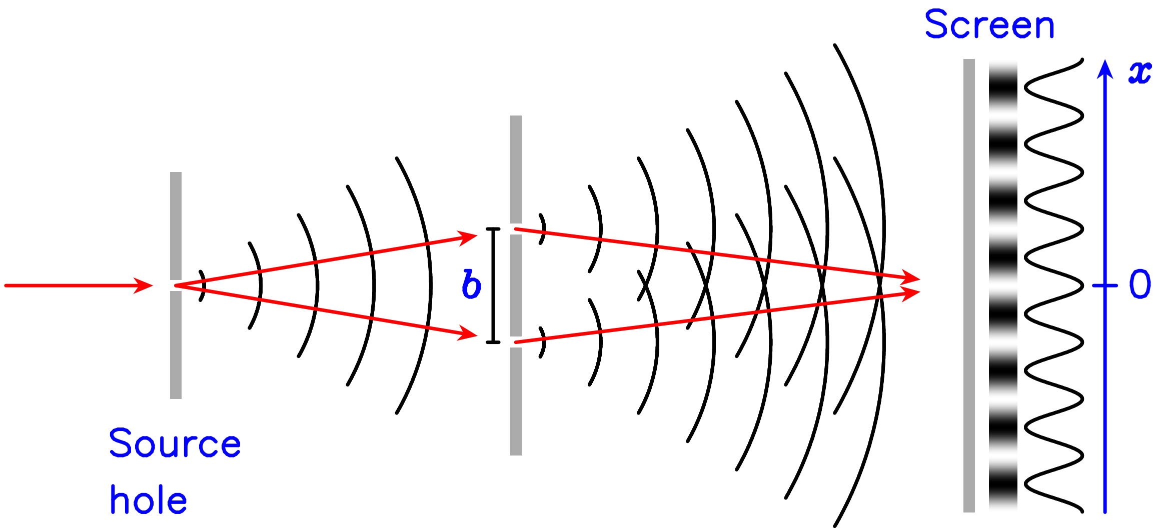
Technically difficult but feasible. $\Rightarrow$ Interferometers!!

This method was developed in the 1950s in England and Australia. Martin Ryle (Univ. of Cambridge) earned a Nobel Prize for his contributions.

Very Large Array, each with a 25 meter in diameter! Now $\theta \sim \lambda/D \sim 1$ arcsec at 45 GHz with D being the Array size (max. baseline ~ 1 km)!

Young's Double-Slit Experiment

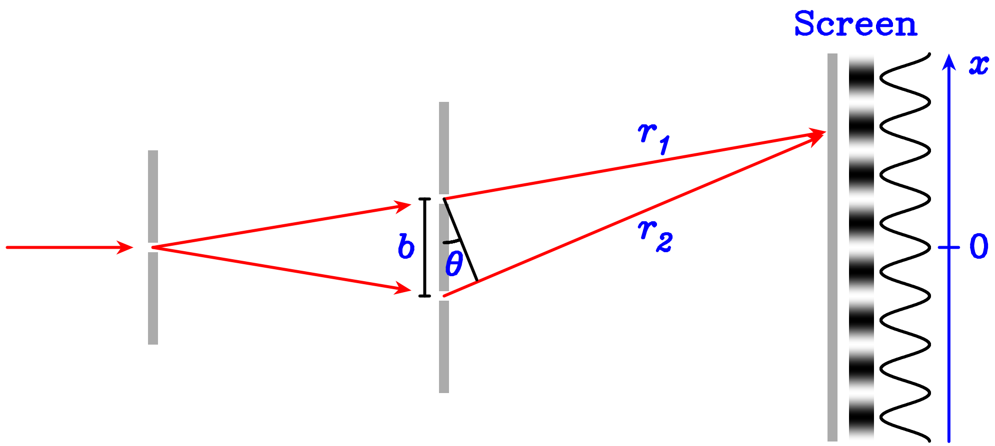
The classic experiment of interference effects in light waves in OPTICAL:



Here assume a Monochromatic source and the rays are in phase when they pass through the slits.

Also the slit width $a \ll \lambda$, so that the slits behave essentially like point sources.

Young's Double-Slit Experiment



At the screen, let's have $E_{(1)} = E_1 e^{i(kr_1 - wt)}$ and $E_{(2)} = E_2 e^{i(kr_2 - wt)}$ from the two Young's slits, and thus the total $E = E_{(1)} + E_{(2)}$. Here $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$.

Obtained image of interference: (time averaged) fringes

$$I(x) = \langle E \cdot E^* \rangle = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(x) = 4I\cos^2(\frac{\pi b \sin\theta}{\lambda})$$

with $x = k(r_2 - r_1) = \frac{2\pi}{\lambda} b \sin\theta$ (as $r_2 - r_1 = b\sin\theta$), $I_1 = E_1^2$, $I_2 = E_2^2$, and $I_1 = I_2 = I$. Thus $I(x) = 4I$ for constructive interference (bright spots) and $I(x) = 0$ for destructive interference (dark spots) ==> fringe. The condition for constructive interference at the screen is $b\sin\theta = m\lambda, m = 0, \pm1, \pm2, \ldots$ Fringe separation (i.e., resolution) $\sim \lambda/(b\cos\theta)$

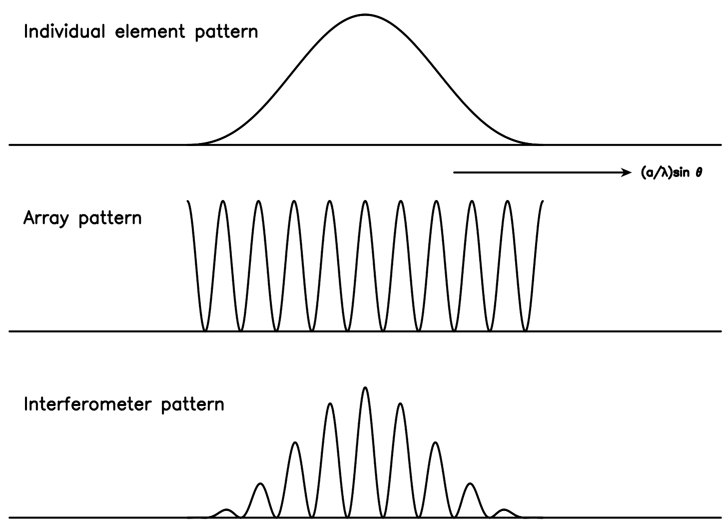
For a point source

What if the slit width is $a \gg \lambda$? Each slit has a Airy diffraction pattern, ie.,

$$P_n(\theta) = \text{sinc}^2(\frac{a \sin\theta}{\lambda})$$

Then combining the effect of the slit width, we have for a point source:

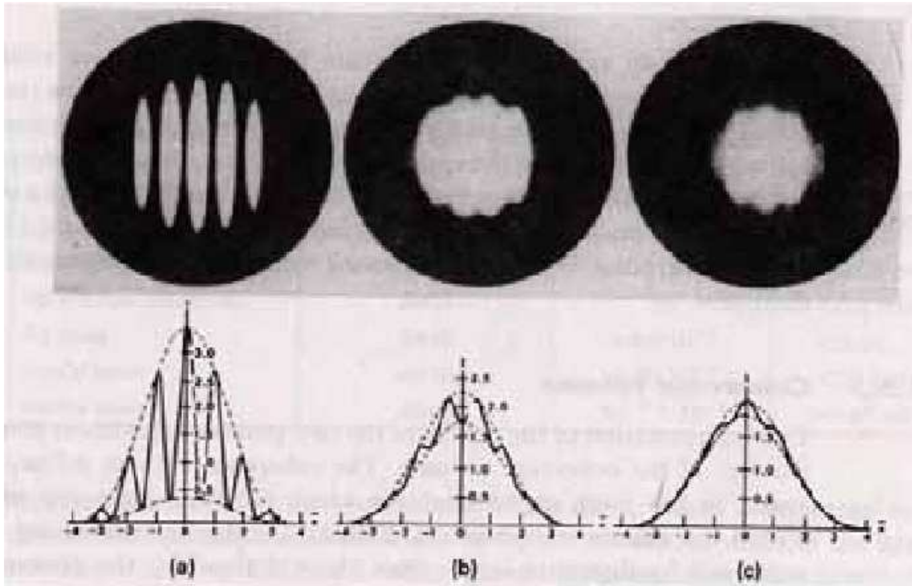
$$I(\theta) = (\frac{a}{\lambda})^2 \text{sinc}^2(\frac{a \sin\theta}{\lambda}) \cdot 4I \cos^2(\frac{\pi b \sin\theta}{\lambda})$$



Here $b = 5a$ and we only show the pattern inside the main lobe of the Airy pattern.

Effect of Source Size

Effect of increasing source size: If the source hole size increases to λ/b, then the fringes will disappear.



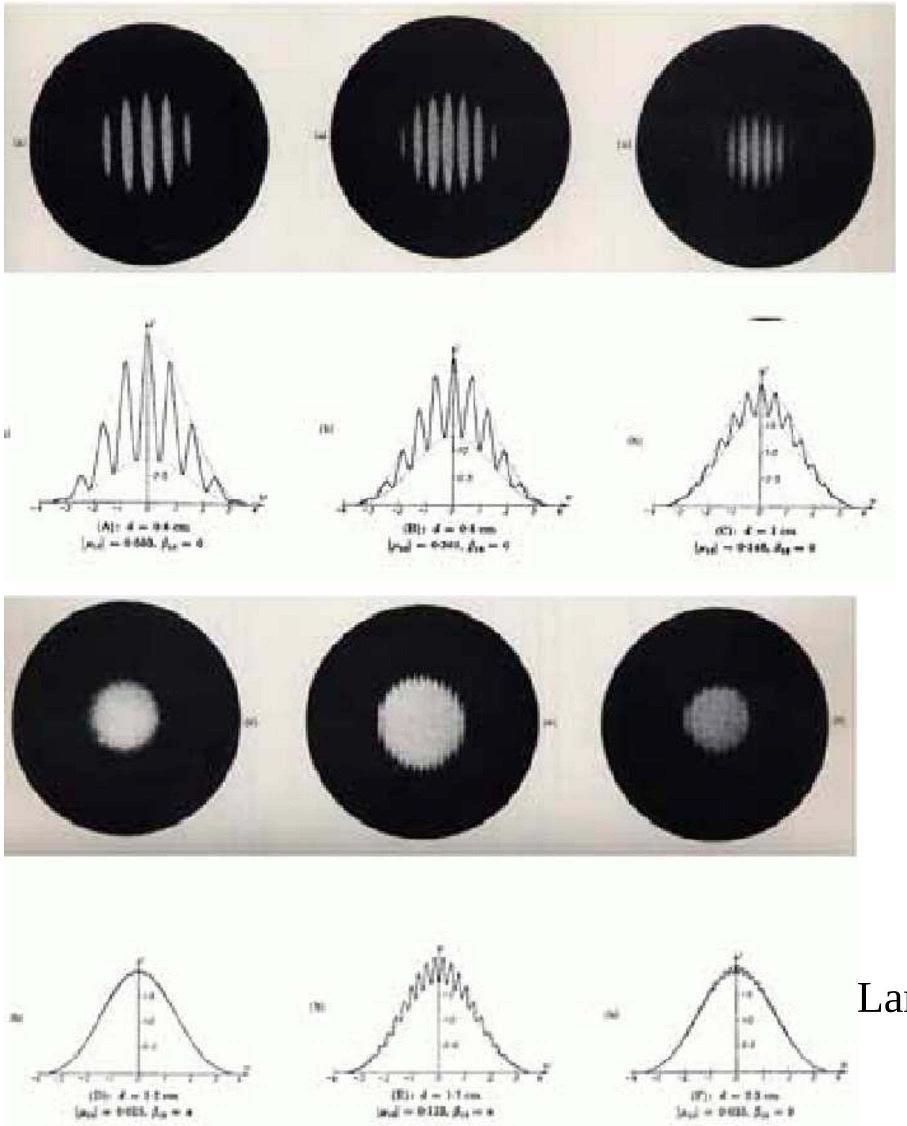
Fringes disappear! $\Rightarrow$ Fringe contrast is linked to the spatial properties of the source.

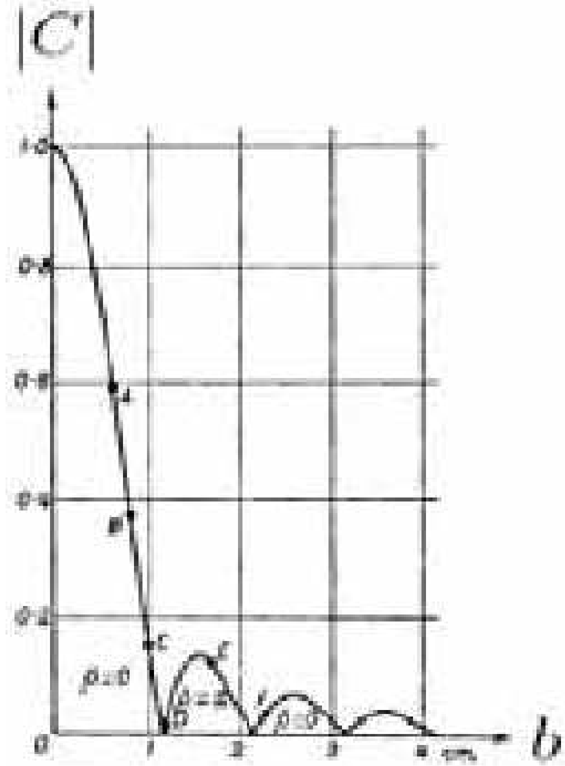
$$I(x) \propto \mathrm{sinc}^2(\frac{a\sin\theta}{\lambda})\Big[I_1 + I_2 + 2\sqrt{I_1 I_2}|C|\cos(x)\Big] \quad \text{with} \quad |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Here $|C|$ a.k.a fringe visibility measured in optical interferometry, a measure of coherency. It is 1 for a coherent source and 0 for not. Here, the coherency is lost as the source size increases, due to the superposition of waves from different (assumed to be incoherent) part of the source.

Effect of separation

Effect of increasing b: The source is a uniform disk





Constructive and destructive interferences!

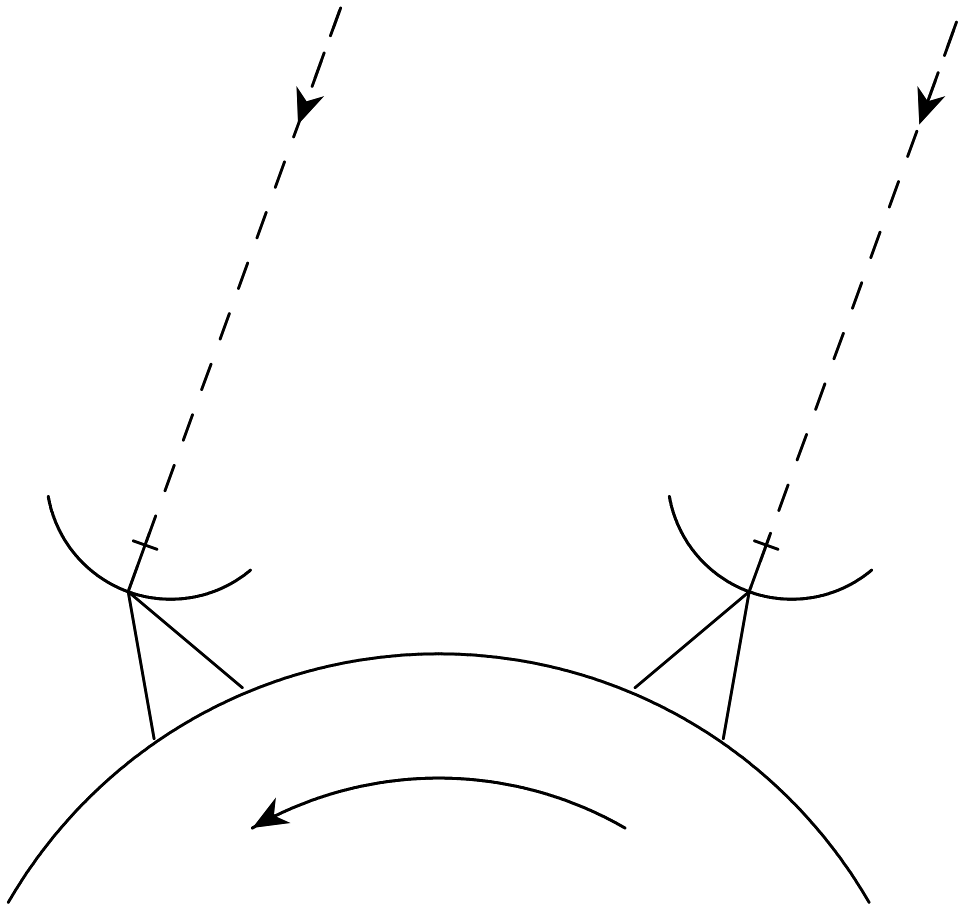
The Visibility is a Fourier Transform of the source!

Measure visibility at different $b \Rightarrow$ source sizes!

Larger b, higher fringe freq., sensitive to smaller scale!

What about the source is a point source?

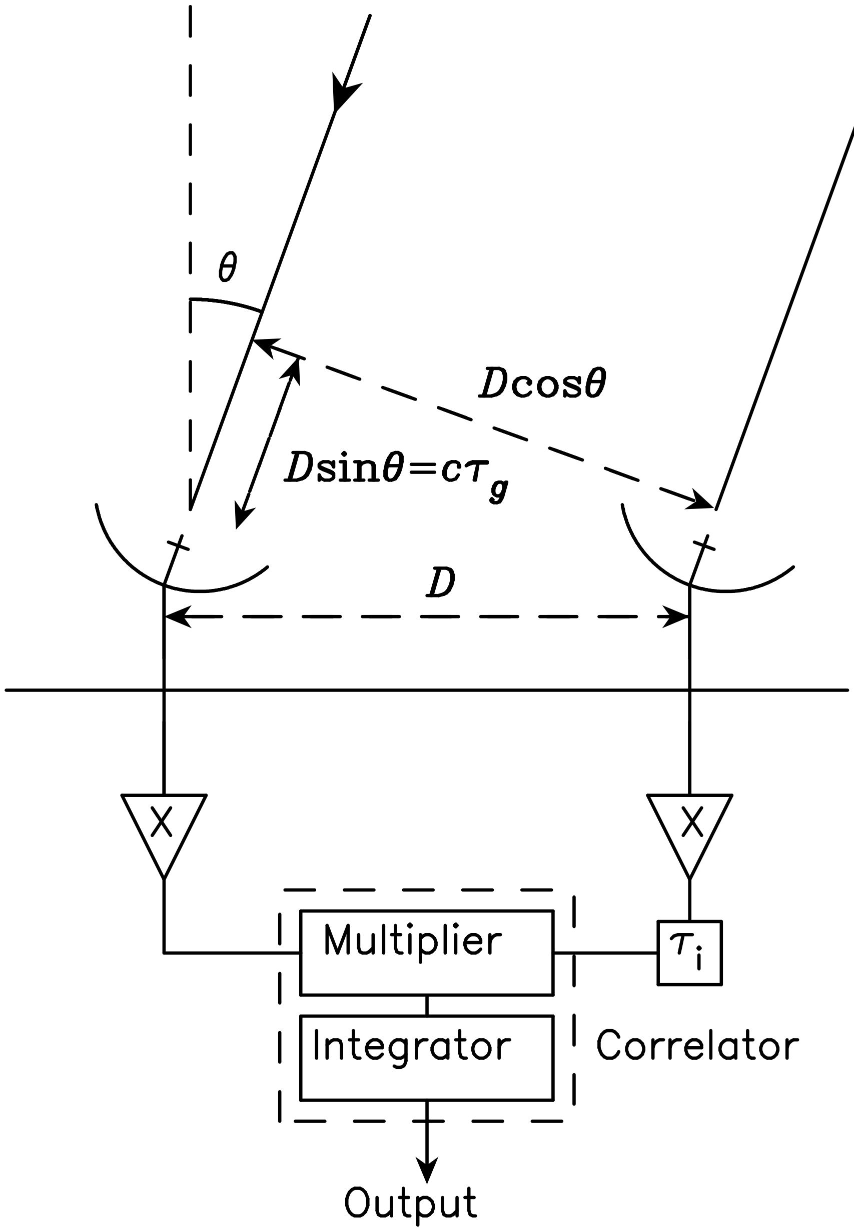
Radio Interferometer



Elementary interferometer. The arrow indicates the rotation of the earth. Here antennas $\Rightarrow$ Young's slits!

An interferometer of n antennas has $\frac{n(n-1)}{2}$ unique pairs of these two-antenna interferometer.

Radio Interferometer



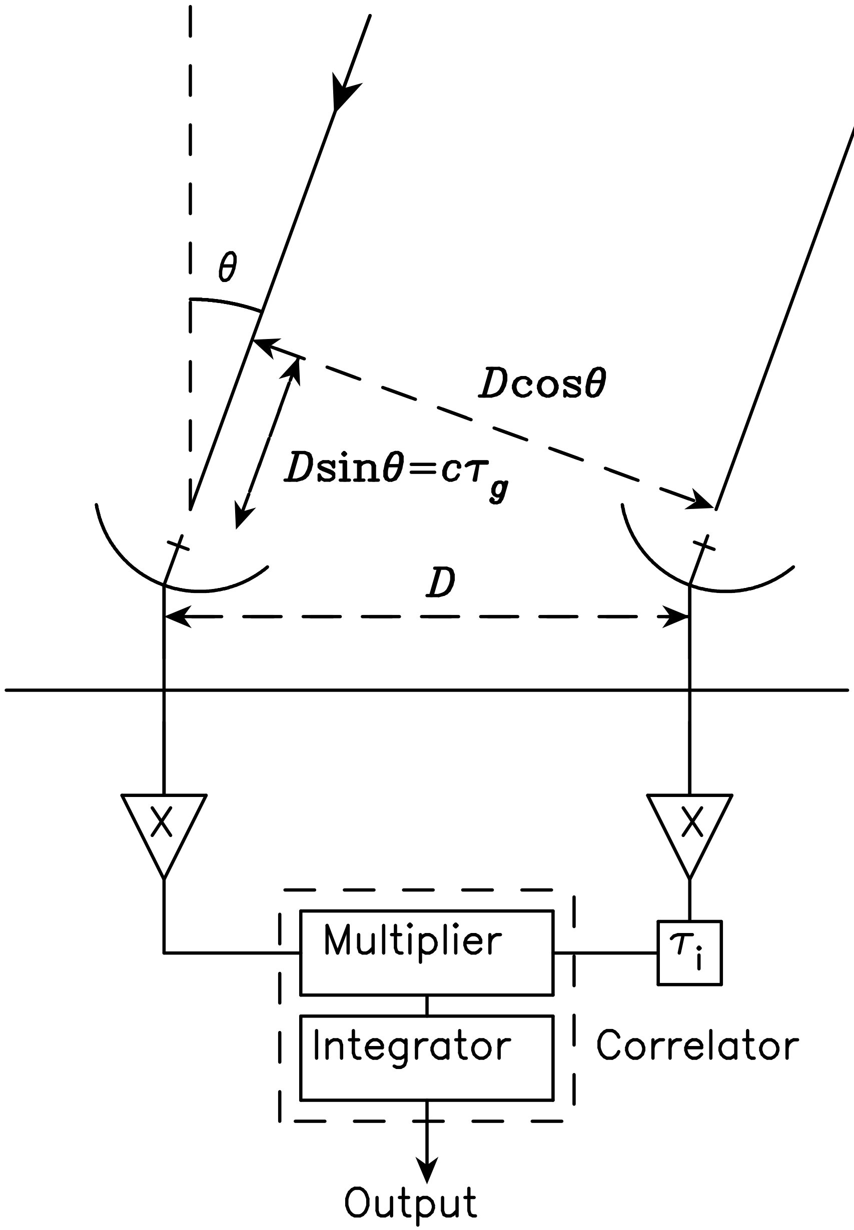
- D: baseline length
- θ: angle of the pointing direction from the zenith, changing with earth rotation.
- τ_g: geometrical delay $= \frac{D}{c} \sin\theta$

The wavefront from the source in direction θ is essentially planar because of great distance traveled, and it reaches the right-hand antenna at a time τ_g before it reaches the left-hand one:

Right: $E\cos(2\pi\nu(t-\tau_g))$ Left: $E\cos(2\pi\nu t)$

The projected length of the baseline on the sky, $D\cos\theta$, changes as the earth rotates.

Radio Interferometer



- τ_g: Geometrical time delay
- τ_i: Instrumental time delay
- X: Bandpass amplifiers.
- Correlator: Multiplier + Integrator
- Output: Fringe Visibility

Multiplier:
Multiply the signals from the two antennas.
$\Rightarrow$ Signals are combined by pairs!

Integrator:
Time Averaging Circuit, e.g., 30 sec integration time for each scan in a SMA observation.

Radio Interferometer without τ_i

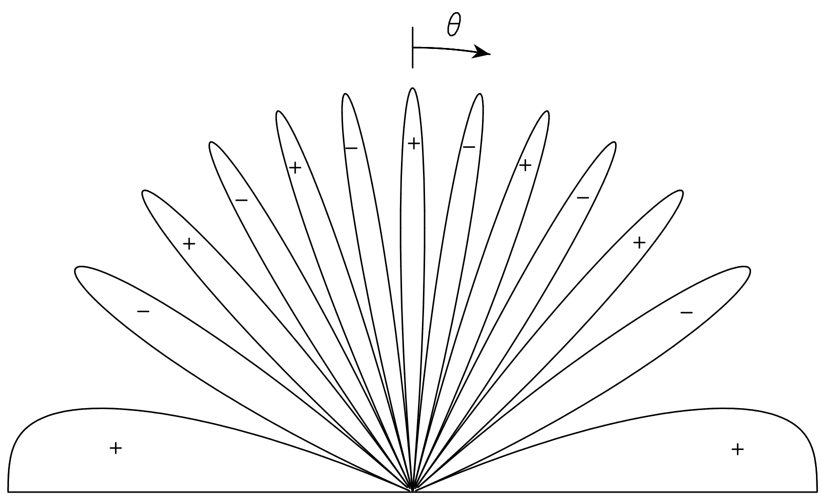
Without any instrumental delay, the output of the multiplier is proportional to

$$F = 2\cos[2\pi\nu t]\ \cos[2\pi\nu(t-\tau_g)] = \cos(2\pi\nu\tau_g) + \cos(4\pi\nu t - 2\pi\nu\tau_g)$$

with $\tau_g = \frac{D}{c}\sin\theta$. Here, due to earth rotation, $\frac{d\tau_g}{dt} = \frac{D}{c}\cos\theta\frac{d\theta}{dt} \ll 1$. With time-averaging integrator, the more rapidly varying term in F is easily filtered out leaving

$$r \equiv \frac{1}{T}\int_0^T F dt = \cos(2\pi\nu\tau_g) = \cos(\frac{2\pi D\xi}{\lambda}) \qquad \text{with} \quad \xi = \sin\theta$$

For sidereal sources the variation of θ with time as the earth rotates generates quasi-sinusoidal fringes at the correlator due to the variation of τ_g.



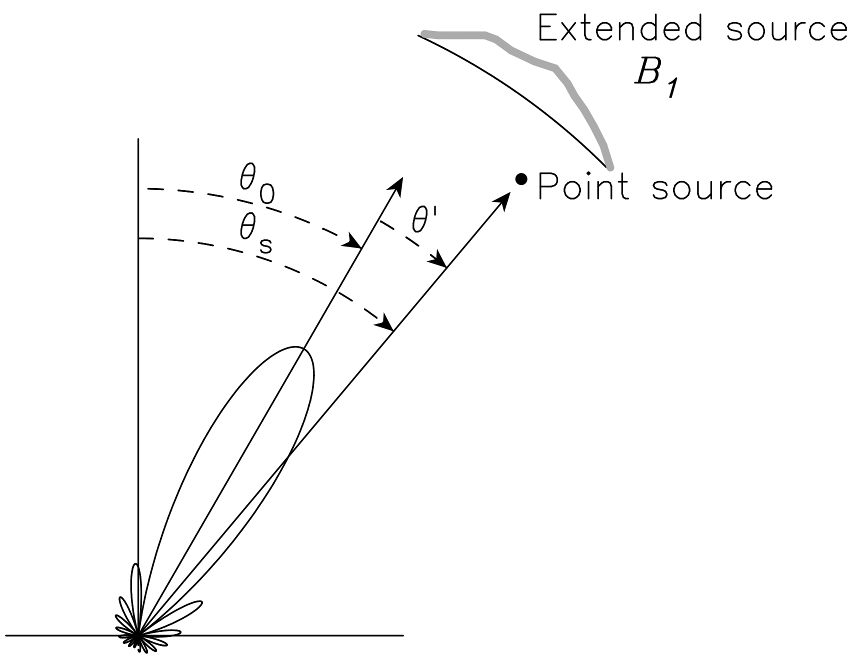
Polar plot of the fringe function r with $D/\lambda = 3$. $\Rightarrow$ Need Fringe stopping by introducing τ_i.

Radio Interferometer with τ_i

Introducing a delay τ_i with extra cable, then the response in the output of the correlator is

$$r = \cos(2\pi\nu\tau) \qquad \text{with} \;\; \tau = \tau_g - \tau_i$$

Now let θ_0 be the pointing center (i.e. pointing direction) of the antenna. Consider a point source at position $\theta_s = \theta_0 + \theta'$, where θ' is very small.



Then with $\tau_g = \frac{D}{c}\sin(\theta_0 + \theta')$, the response of the point source is

$$r = \cos\{2\pi\nu[\frac{D}{c}\sin(\theta_0 + \theta') - \tau_i]\} = \cos\{2\pi\nu[\frac{D}{c}(\sin\theta_0\cos\theta' + \cos\theta_0\sin\theta') - \tau_i]\}$$

Radio Interferometer with τ_i

From the previous slide, we have the response of the correlator (for small θ', we have $\cos\theta' \approx 1$):

$$r \approx \cos\{2\pi\nu[\frac{D}{c}(\sin\theta_0 + \cos\theta_0 \sin\theta') - \tau_i]\}$$

Setting $\tau_i = \tau_g(\theta_0) = (D/c)\sin\theta_0$, then the response is also a sinusoidal fringe pattern:

$$r = \cos(2\pi\nu\frac{D}{c}\cos\theta_0 \sin\theta') \equiv \cos(2\pi u\xi')$$

but with $\xi' = \sin\theta' \approx \theta' = \theta_s - \theta_0$ and

$$u = \frac{D\nu}{c}\cos\theta_0 = \frac{D\cos\theta_0}{\lambda}$$

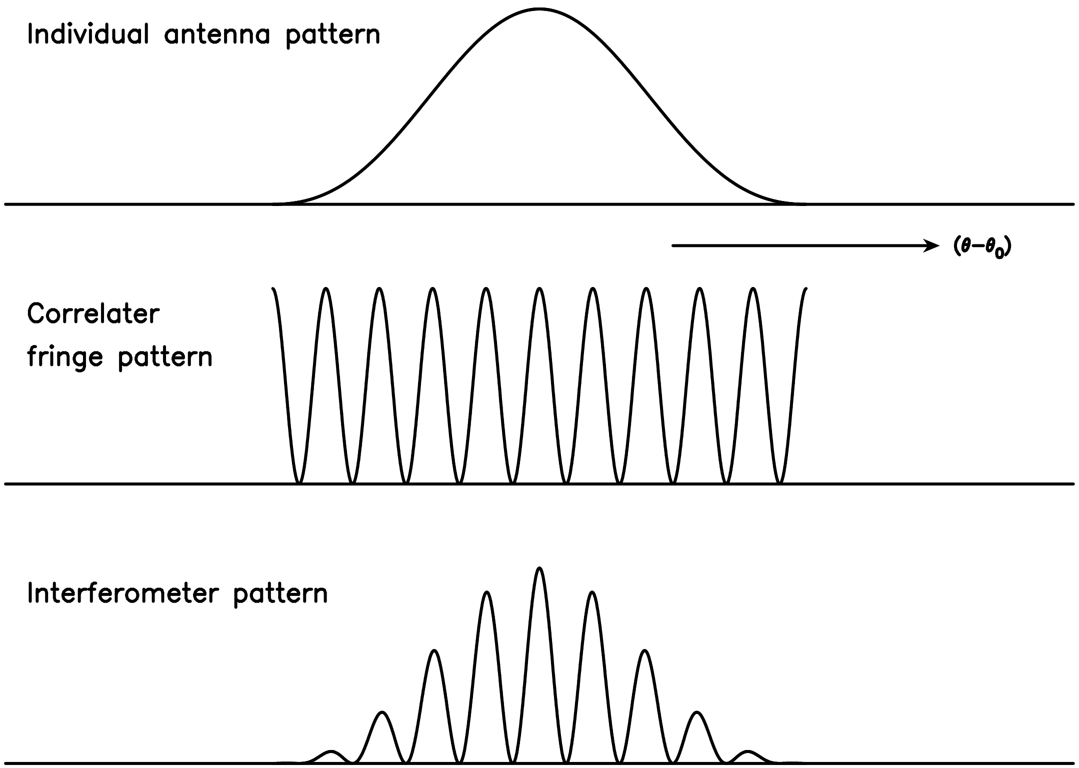
Introducing τ_i, then $\theta \Rightarrow \theta'$, i.e, absolute position $\Rightarrow$ relative position and the fringe pattern is attached to the pointing center. Here u can be considered as a spatial frequency of the fringe defined by the projected baseline in the unit of λ. Also, since u changes with the earth rotation, we can map the source at different u if the antennas track the source during observation. The resolution is then given by $1/u$.

Interferometer

The power reception pattern of an interferometer is the antenna reception power pattern (A_1) multiplied by the correlator fringe pattern r:

$$r_i(u, \theta) = \cos[2\pi u(\theta - \theta_0)]A_1(\theta - \theta_0)$$

A_1 is now called the primary beam pattern (i.e., the field of view) of the antenna.



Visibility function

For an extended source of brightness $B_1(\theta_s)$, the output power of the interferometer is

$$R(u, \theta_0) \propto \triangle\nu \int_{\mathrm{source}} B_1(\theta_s) \cos[2\pi u(\theta_s - \theta_0)] A_1(\theta_s - \theta_0) d\theta_s$$

Since the antennas always track the source and the field of view is small (i.e., < 10 arcmin), we can define $\xi' = \theta_s - \theta_0$ and $B_1(\theta_s) \rightarrow B(\xi')$. So,

$$R(u, \theta_0) \rightarrow R(u) = \triangle\nu \int_{\mathrm{source}} B(\xi') \cos(2\pi u \xi') A_1(\xi') d\xi'$$

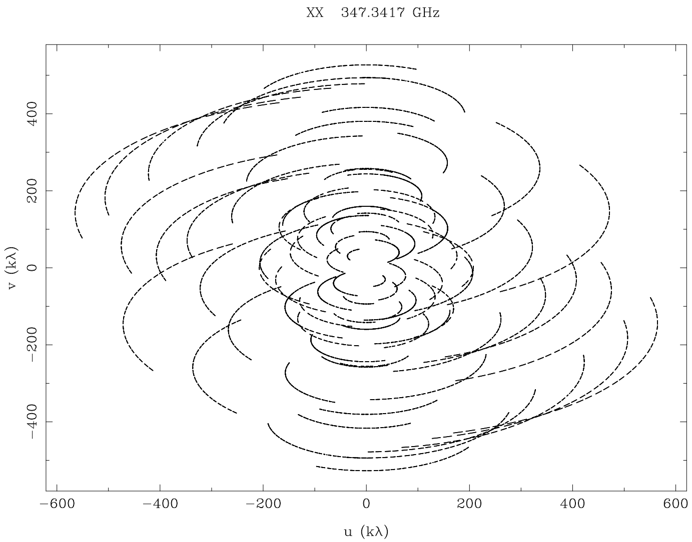
==> Fourier Transform of the source brightness distribution! => Visibility!

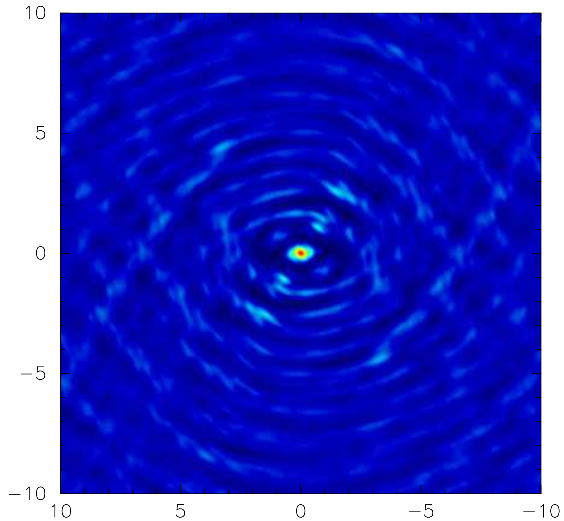
Example: SMA

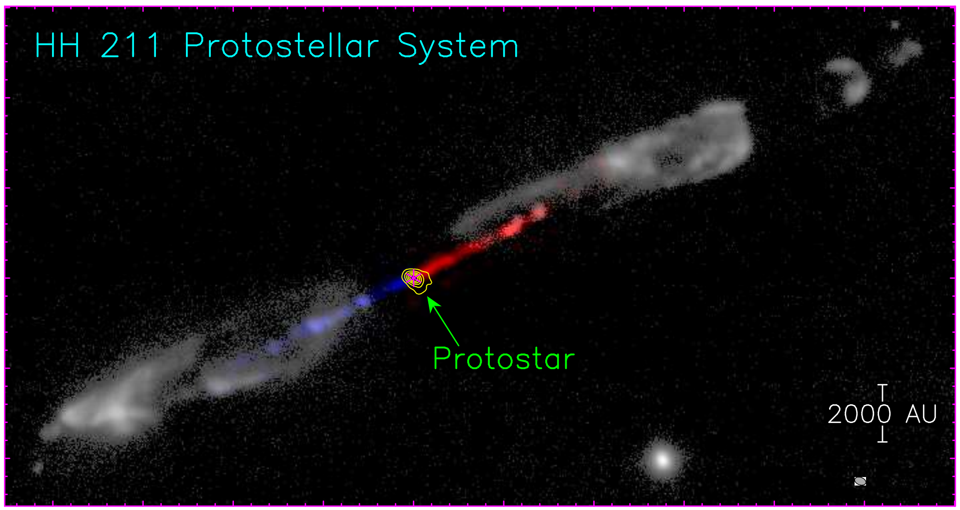
Submillimeter Array: Number of antennas: $n = 8$, number of unique baselines (distance between pairs of antennas): $n(n-1)/2 = 28$.

How about ALMA, which has 16 antennas in the Early Science phase, and 50 antennas in the full operation mode?

Example: SMA HH 211 Jet







==> Resolution given by the synthesized beam fitted to the main lobe of the dirty beam ($\sim 1/u_{\rm max}$)

==> Source image given by inverse Fourier Transform of the Visibility (Next Lecture)