Aperture Synthesis and Imaging

Recommended readings:

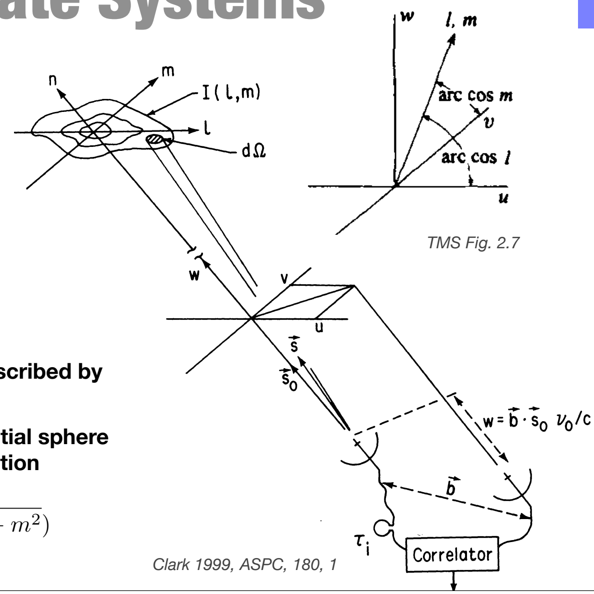
Clark 1999, ASPC, 180, 1; Thompson 1999, ASPC, 180, 11

December 18, 2010 @ ALMA Novice User Workshop

Coordinate Systems

- **Measurements described by** $\lambda(u, v, w)$
- **Radiation on celestial sphere described by direction cosines**

  $(l, m, \sqrt{1 - l^2 - m^2})$



Clark 1999, ASPC, 180, 1

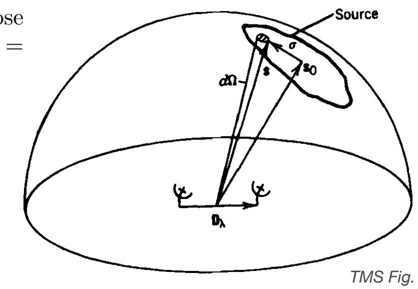
Spatial Coherence Function III

Given the direction cosines, we choose $\boldsymbol{s} = (l, m, \sqrt{1 - l^2 - m^2})$ and $\boldsymbol{s_0} = (0, 0, 1)$ so that

$$\frac{\nu \boldsymbol{s} \cdot (\boldsymbol{r_1} - \boldsymbol{r_2})}{c} = ul + vm + wn,$$

$$\frac{\nu \boldsymbol{s_0} \cdot (\boldsymbol{r_1} - \boldsymbol{r_2})}{c} = w$$

$$d\Omega = \frac{dl\, dm}{n} = \frac{dl\, dm}{\sqrt{1 - l^2 - m^2}}.$$



TMS Fig. 3.1

Substituting the above relations, we find the spatial coherence function to be

$$V_\nu(u, v, w) = \int\int I(l, m)\, e^{-2\pi i[ul + vm + w(\sqrt{1-l^2-m^2}-1)]}\, \frac{dl\, dm}{\sqrt{1 - l^2 - m^2}},$$

where the integral is taken to be zero for $l^2 + m^2 \geq 1$.

Spatial Coherence Function IV

Coplanar Arrays. The first special case consider making all the measurements in a plane, i.e. $\boldsymbol{r_1} - \boldsymbol{r_2} = \lambda(u, v, w = 0)$. The spatial coherence function will take the form

$$V_\nu(u, v, w = 0) = \int\int I_\nu(l, m)\, \frac{e^{-2\pi i(ul + vm)}}{\sqrt{1 - l^2 - m^2}}\, dl\, dm.$$

Sources in a small patch of sky. The second special case consider all the radiation of interest comes from only a small portion of the celestial sphere, i.e. $\boldsymbol{s} = \boldsymbol{s_0} + \boldsymbol{\sigma}$ with $\boldsymbol{s_0} \cdot \boldsymbol{\sigma} = 0$. In other words, $|l|$ and $|m|$ are small that $(\sqrt{1 - l^2 - m^2} - 1)w \simeq 0$ and the spatial coherence function becomes

$$V_\nu(u, v) = \int\int I_\nu(l, m) e^{-2\pi i(ul + vm)}\, dl\, dm,$$

where $V_\nu(u, v)$ is the coherence function relative to the **phase tracking center**, $\boldsymbol{s_0}$.

Effect of Discrete Sampling

Given the above relationship between $V_\nu(u,v)$ and $I_\nu(l,m)$, it is obvious that the direct inversion reads

$$\boxed{I_\nu(l,m) = \int\int V_\nu(u,v)\, e^{2\pi i(ul+vm)}\, \mathrm{d}u\, \mathrm{d}v.}$$

In practice, V_ν is not known everywhere but is sampled at particular places on the $u-v$ plane described by a **sampling function**, $S(u,v)$, that $S(u,v) = 0$ where no data have been taken. One can compute

$$I_\nu^D(l,m) = \int\int V_\nu(u,v) S(u,v)\, e^{2\pi i(ul+vm)} \mathrm{d}u\, \mathrm{d}v,$$

where $I_\nu^D(l,m)$ is referred to as the **dirty image**; its relation to the ideal intensity distribution is

$$I_\nu^D = I_\nu \otimes B,$$

where $B(l,m)$ is the so-called **synthesized beam** or point spread function

$$B(l,m) = \int\int S(u,v)\, e^{2\pi i(ul+vm)}\, \mathrm{d}u\, \mathrm{d}v.$$

Response of Antenna

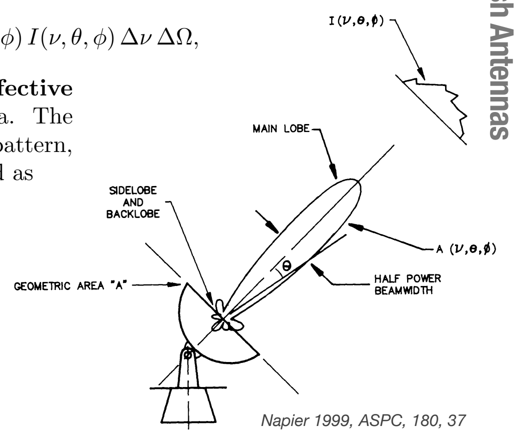
When an antenna is pointed at a source with intensity distribution described by $I(\nu,\theta,\phi)$, the power P received by the antenna in bandwidth $\Delta\nu$ from element $\Delta\Omega$ of solid angle is given by

$$P = A(\nu,\theta,\phi)\, I(\nu,\theta,\phi)\, \Delta\nu\, \Delta\Omega,$$

where $A(\nu,\theta,\phi)$ (m^2) is the **effective collecting area** of the antenna. The normalized antenna reception pattern, A, or **power pattern** is defined as

$$\mathcal{A}(\nu,\theta,\phi) = A(\nu,\theta,\phi)/A_0,$$

where A_0 (m^2) is the response at the center of the main lobe and is equivalent to the **effective area** of the antenna.



Napier 1999, ASPC, 180, 37

Effect of Primary Beam

In practice, the interferometer elements are not point probes which sense the voltage at that point, but are elements of finite size and directional sensitivity. The normalized reception pattern of each element, i.e. the **primary beam** needs to be included

$$V_\nu(u,v) = \int\int \mathcal{A}_\nu(l,m) I_\nu(l,m)\, e^{-2\pi i(ul+vm)}\, \mathrm{d}l\, \mathrm{d}m,$$

where $\mathcal{A}_\nu(l,m) = A_\nu(l,m)/A_{\nu,0}$.

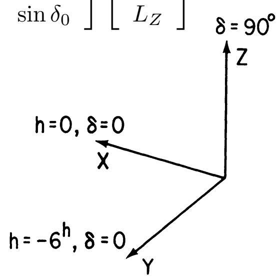
The calibration with the element directional sensitivity $\mathcal{A}_\nu$ should be postpoined to the final step of deriving the sky intensity distribution and it should simply divide the derived intensities. Such division is often referred to as **primary beam correction** and will, however, not only produce a better estimate of the actual intensities in this direction, but will also increase the errors in directions far from the phase tracking center.

Projection of Baselines I

With multi-element arrays, it is convenient to specify the antenna positions in a Cartesian coordinate system. For example, a system with X the direction of the meridian at the celestial equation, Y the East, and Z toward the North celestial pole. Let L_X, L_Y, and L_Z the corresponding coordinate differences for two antennas, the baseline components (u,v,w) are given by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{bmatrix} \begin{bmatrix} L_X \\ L_Y \\ L_Z \end{bmatrix},$$

where H_0 and δ_0 are the hour angle adn declination of the phase reference position. The elements in the transformation matrix are the direction cosines of the (u,v,w) axes relative to (X,Y,Z) axes.



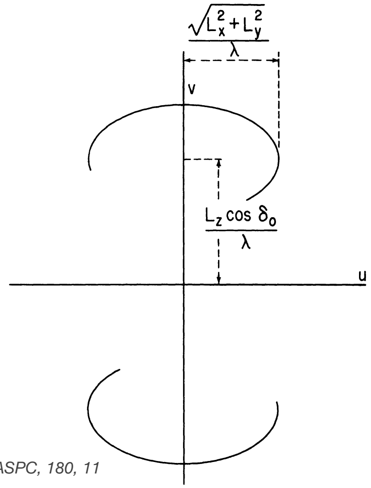
Thompson 1999, ASPC, 180, 11

Projection of Baselines II

Eliminating H_0 from the expressions for u and v, we can obtain the equation of an ellipse in the (u, v) plane:

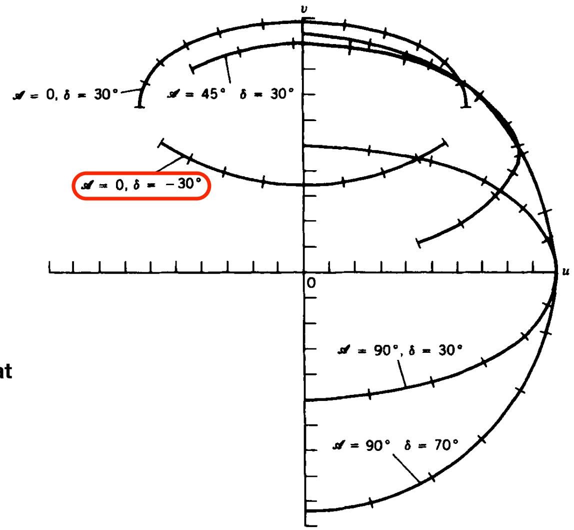
$$u^2 + \left(\frac{v - (L_z/\lambda)\cos\delta_0}{\sin\delta_0}\right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}.$$

The ellipse is simply the projection onto the (u, v) plane of the circular locus traced out by the tip of the baseline vector. Since $I(l, m)$ is real, $V(-u, -v) = V^*(u, v)$. For an array of antennas, the ensemble of elliptical loci is the **sampling function**, $S(u, v)$.



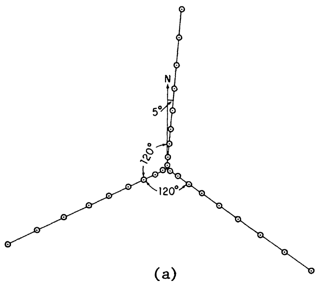
Thompson 1999, ASPC, 180, 11

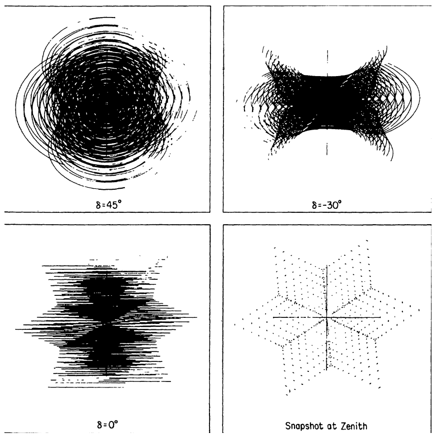
Projection of Baselines III



- **Loci for an array at latitude +40°**

VLA u-v Tracks





Thompson 1999, ASPC, 180, 11

From Visibility to Images

Observed quantities:

Visibilities $$V(u,v)S(u,v) = \int\!\!\int \mathcal{A}(l,m) I(l,m)\, e^{-2\pi i(ul+vm)} \mathrm{d}l\, \mathrm{d}m$$

Sampling function $$S(u,v) = \sum_{k=1}^{N} \delta(u - u_k, v - v_k)$$

Generating images from visibilities:

Synthesized beam $$B(l,m) = \int\!\!\int S(u,v)\, e^{2\pi i(ul+vm)} \mathrm{d}u\, \mathrm{d}v$$

Dirty map $$I^D(l,m) = [\mathcal{A}(l,m) I(l,m)] \otimes B(l,m) = \int\!\!\int V(u,v) S(u,v)\, e^{2\pi i(ul+vm)} \mathrm{d}u\, \mathrm{d}v$$

Weighted visibilities $$V^W(u,v) = \sum_{k=1}^{N} R_k T_k D_k V(u,v),$$

where R_k, T_k, and D_k are weights assigned to the visibilities indicating their reliability, the tapering, and the density weighting.

Gain Calibration in Miriad

calibrator visibilities

deriving gains (selfcal)

gain table

transferring gains (gpcopy)

calibrated calibrator visibilities

calibrated source visibilities

Imaging with Miriad

calibrated visibilities

sampling function

FFT (invert)

dirty map

synthesized beam

deconvolution (clean)

model map

residual map

convolution (restor)

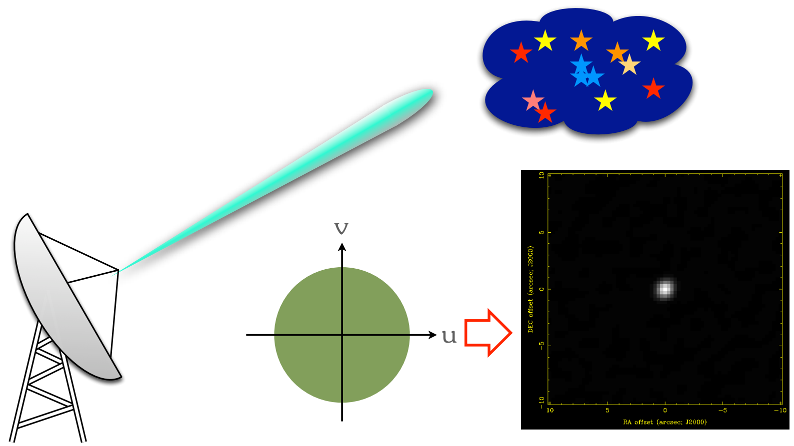
Gaussian fit of the beam

clean map

residual map

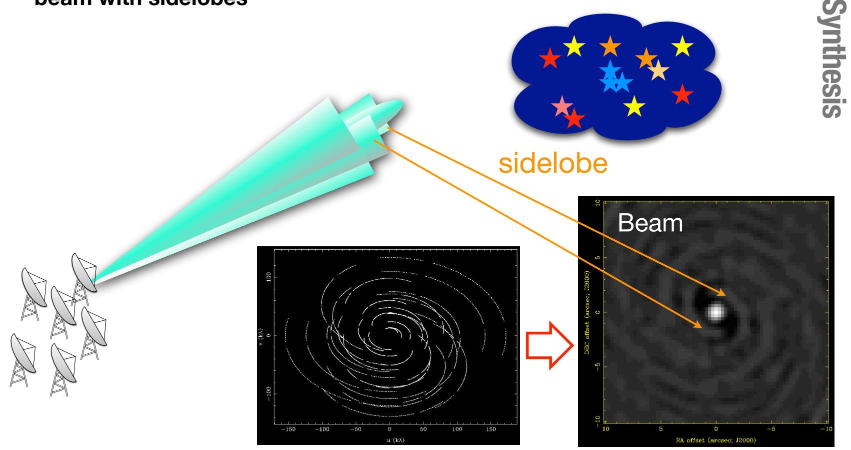
Why Restoring a Beam?

- Single-dish comes with a more or less ideal beam



Why Restoring a Beam?

- Incomplete sampling with an array leads to a non-ideal beam with sidelobes



Replacing Synthesized Beam

dirty map = sky brightness ⊗ synthesized beam

deconvolution (clean) means dirty map, synthesized beam ⇨ model map ≃ sky brightness

residual map ≃ error map

residual map = dirty map − model map ⊗ beam

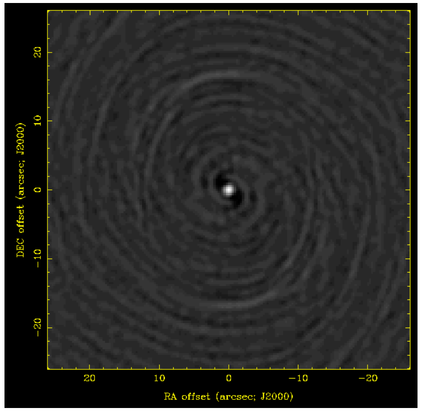
convolution (restor) means

synthesized beam ⇨ Gaussian beam

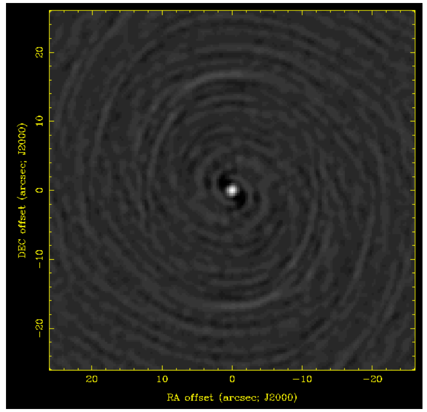
clean map = model map ⊗ Gaussian beam + residual map

Calibrator 0359+509

Dirty map of 0359+509



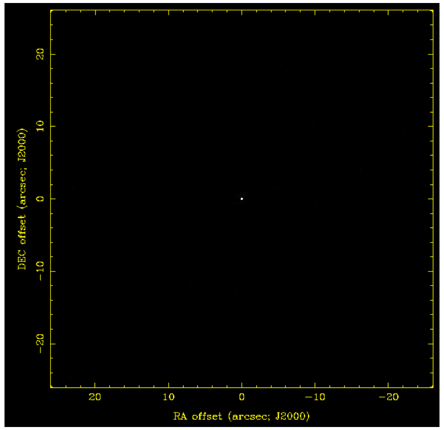
Beam of 0359+509



Model Map of 0359+509

- Appear to be a point source
  - Gain calibration okay

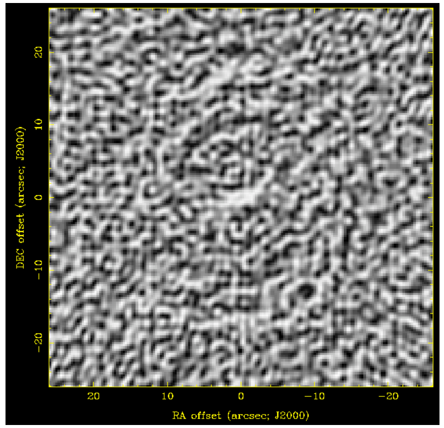
Model map of 0359+509



Residual Map of 0359+509

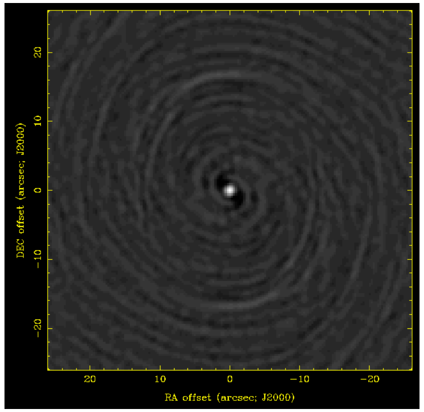
- No more significant feature
  - Deconvolution okay

Residual map of 0359+509

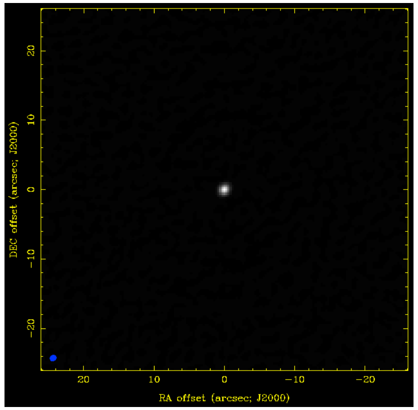


Clean Map of 0359+509

Dirty map of 0359+509

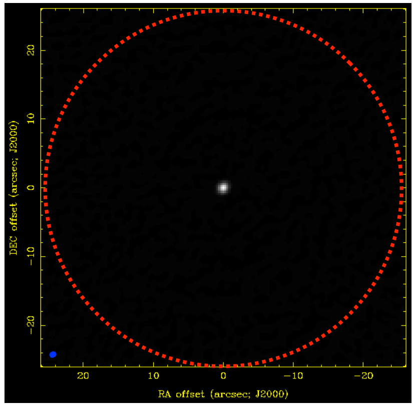


Clean map of 0359+509

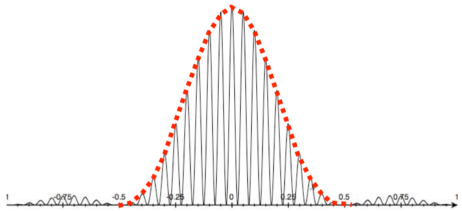


Field of View (FoV)

Clean map of 0359+509

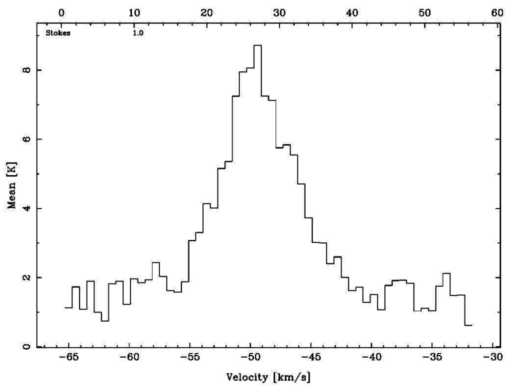


- Analogous to the double-slit fringes modulated by a single-slit response
- Each antenna element of an array also sets field of view



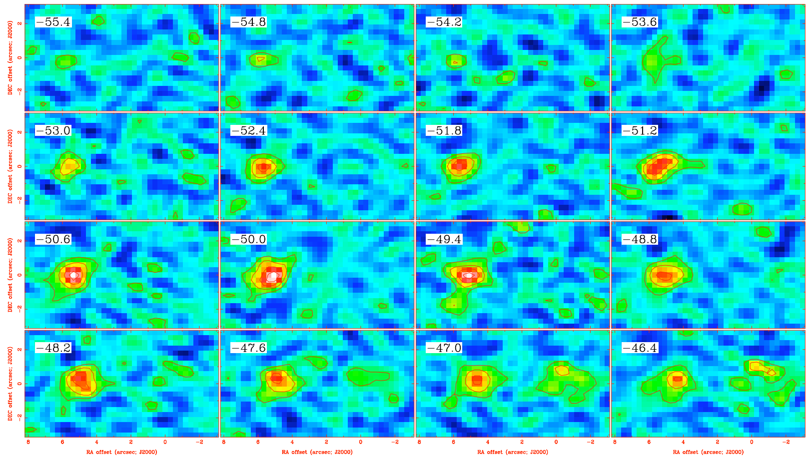
Channel Maps - Data Cube

- **Data cube** (l, m, v)
  - Probe radial velocity through Doppler effect in the 3rd dimension
  - CH_3CN *J*=12-11 *K*=3



Channel Maps

- **Data cube** (l, m, v)
  - Probe radial velocity through Doppler effect in the 3rd dimension



Thanks for Coming...